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# Mathematics



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It is our wish that teachers will use this document to improve their teaching and so better attain the key objectives of Seventh-day Adventist education.

Sincerely

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How Mathematics Shows God's Work in Nature Making Mathematics Practical and Relevant Making Mathematics Fun Problem Solving and Investigation Essential Competencies Remedial Mathematics Mathematics Departmental Policy

# SUGGESTIONS FOR USING THE FRAMEWORK

There are many ways to use the framework to help you plan courses, units of work, or topics. The aim is to provide a basic checklist for planning. For topics or units, try the following steps, summarizing your plan on a page or two.
Read the philosophy, rationale, and objectives on pages 6-11 to gain a picture of the key emphases of your subject.
Check through the list of value-oriented concepts on page 14. An example is 'integrity'. Select those that seem to need most emphasis, and write them down.
Read the suggestions on how to teach values on page 15 - 23. An example is making value judgments. Now add some ideas on teaching methods to your summary.
From pages 16 to 23, you will find suggestions for using the value-concepts. Use appropriate ideas from this section and your own creativity in your summary.
Refer to the set of skills on page 25, and select those that are applicable. An example is 'interpretation'. There need not be numerous skills. Write these down.
For ideas about assessment, check page 26. Assessment must be linked with your teaching content, emphasis and methods. Jot down some ideas.
To see how value-concepts, skills and teaching methods can be worked into planning, look at the topic summary on page 27 and the unit overview flowchart on page 29. You can now refine your own summary overview page of your topic or unit.
To make your overall planning more systematic, it could be helpful to use the correlation charts on pages 30 - 32. See the instructions for using these on page 30.
Refer to the appendices for further teaching resources. As you can see, you are required to thoughtfully bring together a number of strands in the planning process. By this stage, the topic or unit is certain to be taking on a values-oriented Adventist Christian perspective.

# WHAT IS A FRAMEWORK?

In the Adventist school context, a 'framework' is a statement of values and principles which guide curriculum development. These values and principles are derived from Adventist educational philosophy which states important ideas about what is real, true and good. The objective of the framework is to show how Adventist Christian values and faith can be integrated with academic learning, and it provides some examples of how this can be done. The framework is organized as a resource of ideas for subject planning, so it is intended to be useful rather than exhaustive.

In attempting to present an Adventist perspective, it is clearly understood that some aspects of a course may be taught in similar fashion no matter where it is taught. However at a superficial level the objectives and content of other topics taught in Adventist schools will seem little different from state syllabi merely because the content appears relatively neutral in philosophical terms. With thoughtful teaching there will in fact be differences in content emphasis and approach for these topics, while there will be more noticeable differences in other topics and curriculum processes. These differences of varying degree will stem from the underlying philosophy of the framework.

This mathematics framework contains: a philosophy; a rationale; aims; objectives; a list of attitudes; a list of values; values teaching strategies; a list of skills and processes; guidelines on assessment; a unit summary; a unit flowchart; concept correlation charts; and appendices.

In the framework there is acceptance of the requirement to teach the basic skills, topics and thematic links outlined in the various state syllabi. The document sets out some of these in checklists and gives practical suggestions which will be used in the preparation of course outlines, programs, units and lessons. It therefore becomes a basic reference point for teachers. No attempt has been made to replace an individual teacher's personal programs, but rather to guide the development of these programs. Overall, the framework mainly attempts to show how good practice can be placed in a value perspective which is Christian. It is a tool teachers can use to help them either reinterpret state curriculum documents from an Adventist perspective, or to simply teach from that perspective if the subject is an elective one.

# WHO IS THE FRAMEWORK FOR?

The framework is primarily designed for all mathematics teachers in Adventist secondary schools. It also provides a reference point for curriculum planning for principals and administrators in the Adventist educational system. Further, it attempts to show government authorities that there is an Adventist curriculum emphasis which provides some justification for the existence of an Adventist school system. The document is to be used in establishing the direction for any curriculum planning, whether it involve creating courses from scratch, adding to state syllabi, or evaluating units and resources.

# FRAMEWORK OVERVIEW

MATHEMATICS IS DEFINED AS:	<ul> <li>A search for patterns and relationships.</li> <li>A powerful, precise and concise means of communication used to represent, interpret, explain and predict.</li> <li>Creative activity involving invention, intuition and discovery.</li> </ul>
MATHEMATICAL COMPETENCE IS DEVELOPED BY KNOWING:	<ul> <li>Concepts</li> <li>Facts and terms</li> <li>Formulae</li> <li>Relationships</li> <li>Rules and theorems</li> </ul>
BY DEVELOPING ATTITUDES SUCH AS:	<ul> <li>Courage</li> <li>Enjoyment</li> <li>Honesty</li> <li>Neatness</li> <li>Organization</li> </ul>
BY EXPLORING VALUES SUCH AS:	Accuracy     Choice     Creativity     Economy     Pattern
BY DEVELOPING SKILLS SUCH AS:	<ul> <li>Data processing</li> <li>Inquiry</li> <li>Social competency</li> <li>Synthesizing</li> <li>Thinking</li> </ul>
BY USING CONTENT WHICH INCLUDES:	Making mathematics fun     Problem solving     Relevant topics

# **PHILOSOPHY**

Everywhere in nature there are evidences of mathematical relationships. These are shown in ideas of number, form, design and symmetry, and in the constant laws governing the existence and harmonious working of all things. Through its study of these laws, ideas and processes, mathematics can reveal to students some of God's creative attributes, and particularly His constancy.

Mathematics may also develop students' capacity to use appropriate thought processes to more clearly identify aspects of truth which relate to natural laws and design. Such truth is predictable, in that given a set of axioms and the appropriate mathematical processes, the result is always as expected. Therefore when students learn mathematical processes, axioms and laws, they may be further enabled to more clearly identify God's design and handiwork in nature.

While mathematics is a pure science allowing many hypotheses and conjectures to be conclusively demonstrated as being either correct or incorrect, it also opens possibilities of study topics which defy either proof or disproof. Examples are infinite smallness and infinite greatness. This unusual balance between the unexplained and the clearly evident provides for the student an accurate picture of an infinite and eternal God, whom we can neither prove nor disprove, yet in whom we believe. However, God has created rules and functions which can be demonstrated as an evidence of His presence.

"Mathematics is a revelation of the thought life of God. It shows Him to be a God of system, order, and accuracy. He can be depended upon. His logic is certain. By thinking in mathematical terms, therefore, we are actually thinking God's thoughts after Him." 1

Whereas the student cannot comprehend the absolute unchangeable nature of God, mathematical dependability demonstrates clearly the consistency of God and His perfect creation. This is a demonstration of total dependability.

The mathematics program of the school should be an important aid in developing the creativity of the individual. Here the student has limitless opportunity to test his skills against the immutability of God's law. In a very real sense students of mathematics will develop confidence as they examine the consistency of law.

1. (Byrne, Ed.D., A Christian Approach to Education, Mott Media).

# **RATIONALE**

There are many reasons why students should learn mathematics. Firstly, they need to master basic mathematical skills in order to cope with the demands of life. Such demands include being numerically literate, gaining the tools for future employment, developing the prerequisites for further education, and appreciating the relationship between mathematics and technology.

Secondly, mathematics is the language of the sciences, and many disciplines depend on this subject as a symbolic means of communication.

Thirdly, a particularly important life skill is decision-making. Mathematics education can play an important part in developing students' general decision-making and problem solving skills.

A fourth justification for learning mathematics is the need for students to use the subject as an important means of discovering truth. The discipline clearly and precisely presents aspects of knowledge which are helpful in finding out truth about the structure and patterns of the environment, and of some of the ways in which God has communicated with man.

A fifth objective closely associated with the quest for truth, is the search for beauty. Mathematics can assist students to develop their aesthetic aptitudes by looking at patterns in nature and by appreciating the precision and symmetrical beauty in God's creation.

## **AIMS**

## The study of mathematics aims to:

- 1. Develop an awareness that order and precision are characteristic of the Creator.
- 2. Develop a growing knowledge of God's faithfulness and dependability.
- 3. Develop a positive set of attitudes through learning mathematics. Examples are self-discipline, self-confidence, patience, and courage.
- 4. Present mathematics as a living art which is intellectually exciting, aesthetically satisfying, and relevant to a great variety of practical situations.
- 5. Make the students aware of the relevance of mathematics through the study of worthwhile applications.
- 6. Make students competent to handle mathematics in a wide variety of life situations.
- 7. Allow students to appreciate the value of calculating devices in mathematics.
- 8. Develop a positive, adventurous attitude towards mathematics and the learning of mathematics.
- 9. Encourage confidence and provide an environment for the enjoyment of mathematics.
- 10. Develop the skills of logical thinking and presentation.
- 11. Increase the students' range of mathematical concepts and skills, and their ability to use them in solving real problems.
- 12. Maintain the basic skills previously developed.
- 13. Develop the synthesis skills of using techniques from different areas of mathematics to solve a problem.
- 14. Develop skills in talking, listening, reading and writing about mathematics.
- 15. Develop an ability to communicate using the symbolism and procedures of mathematics.
- 16. Support other fields of study which make use of mathematical techniques.
- 17. Enable students to use mathematics in coping with, controlling and determining the factors which will influence their present and future environments.

# **OBJECTIVES**

#### **KNOWLEDGE**

Students studying mathematics should be able to:

- 1. Recall mathematical facts;
- 2. Understand and use mathematical terminology;
- 3. Understand mathematical concepts and relationships;
- 4. Understand the historical contribution of mathematics to society;
- 5. Know relevant formulae, equations, rules and theorems and their proofs when appropriate;
- 6. Know relevant procedures and techniques such as the method of proof by induction:
- 7. Recall basic shapes of the graphs of the functions and relations used;
- 8. Understand where mathematics is used in real life.

#### **PROCESSES**

Students should be able to:

- 1. Access the appropriateness of a particular strategy in solving a problem;
- 2. Identify and execute the discrete steps necessary to solve a range of practical problems;
- 3. Translate realistic written and oral problems into mathematical symbols and vice versa:
- 4. Draw and attempt to justify conclusions or hypothesis in relation to sets of data;
- 5. Make informed decisions based on a mathematical evaluation of various options;
- 6. Access the accuarcy of results in relation to a given context;
- 7. Analyse and interpret data;
- 8. Discover generalisations and express them mathematically.

### **SKILLS**

#### Calculation

Students should be able to:

- 1. Develop manipulative and computational skills;
- 2. Accurately perform calculations, using a calculator where appropriate;

## Calculation (cont'd):

- 3. Read information expressed in mathematical words and symbols;
- 4. Substitute in appropriate formulae;
- 5. Verify the suitability and reasonableness of a result.

## **Data Processing**

## Students should be able to:

- 1. Acquire skills in collecting data from a variety of sources;
- 2. Develop skills in organising information;
- 3. Practise practical methods of summarising and presenting data;
- 4. Show facility in drawing graphs and diagrams;
- 5. Develop systematic ways of recording information.

## Inquiry

#### Students should be able to:

- 1. Develop investigation and inquiry skills;
- 2. Acquire skills in using multiple references, and reading widely but selectively;
- 3. Develop oral and written communication skills, including the ability to use precise terminology;
- 4. Manipulate concrete materials, mathematical instruments and measuring devices;
- 5. Develop initiating strategies seeking patterns, constructing tables, listing.

#### Thinking

#### Students should be able to:

- 1. Translate realistic written and oral problems into mathematical symbols and vice versa;
- 2. Make informed decisions based on a mathematical evaluation of various options;
- 3. Understand the nature and role of inductive and deductive reasoning and proof, and reason inductively and deductively;
- 4. Apply suitable mathematical techniques and problem solving strategies to routine and non-routine situations.

#### Communication

#### Students should be able to:

- 1. Demonstrate basic writing skills;
- 2. Present work with appropriate setting out and neatness;
- 3. Clearly understand instructions and follow them.

#### Social

#### Students should be able to:

- 1. Accept responsibility for their own actions;
- 2. Follow directions;
- 3. Listen and be tolerant of others' views;
- 4. Contribute to group discussion and activities;
- 5. Persevere when problems arise;
- 6. Start work without prompting.

#### **ATTITUDES**

## Students should:

- 1. Develop self-confidence in handling mathematics;
- 2. Develop an appreciation of the value of mathematics in society, and apply this appreciation in their everyday contexts;
- 3. Be willing to experiement mathematically in unfamiliar situations;
- 4. Show a willingness to participate in the learning of mathematics;
- 5. Desire to persevere;
- 6. Strive for a neat, orderly and logical presentation;
- 7. Positively affirm mathematics as being intellectually exciting;
- 8. Interact in a cooperative manner with peers and teachers.

# STUDENT ATTITUDES

Listed below are attitudes which are thought necessary for students to develop if they are to grow in both their mathematical ability and in their ability to cope with life.

## Courage:

• Face mistakes, give answers in front of class when unsure, to ask questions or ask for help, persistence, courage to question teacher or book answers.

## **Enjoyment:**

 When success is achieved, in teachers' approach to mathematics, and with pleasantly decorated room.

## Honesty:

- Educate them to use answers wisely as learning experiences;
- Refrain from copying from others.

#### Learn from mistakes:

- Realise that all people make mistakes, including text books, and that mistakes show students' weaknesses and strengths in topics;
- For brighter students, mistakes show carelessness. Text corrections are very important.

## Mutual cooperation:

- More able students can help others;
- Also shown in student teacher relationships.

#### Neatness:

• Attention to detail, taking time with diagrams; clearly defining answers; accuracy in use of symbols and signs, no liquid paper; crossing out with single line.

## Organisation:

• Folders, notes, tests, corrections, time. Discourage wasting time with undue embellishments such as changing colours.

#### Respect for self and others:

- Every one has ability and is able to contribute in some way and can be considered useful;
- Tolerance for those who differ race, religion, ability, beliefs, ideas, ways of doing things. Do not laugh at the mistakes of others;
- Disapprove derogatory statements about lower maths levels and those of lower ability.

#### Self-confidence:

- Have students experience success at beginning of unit in particular;
- Emphasize that making a mistake does not mean failure;
- Attempt problem solving in varying circumstances;
- Attitude of teacher toward student avoid put downs;
- Lower ability students can achieve in certain areas. An example is tesselations.

## Self-discipline:

• Encourage students to do homework because of personal benefits, doing as much as they can rather than as little as they can. Show the wise use of answers, and benefits of making personal sacrifices, patience, and emphasize patience particularly if success is delayed.

## Sense of justice:

- Fairness must be seen in discipline and marking;
- Teacher needs to admit a mistake and apologise if necessary.

#### Time:

- Emphasize efficiency of time, quality time. (There needs to be short periods of concentrated effort, and undisturbed time);
- Teachers return tests in reasonable time;
- · Allow planned study and revision time.
- By example, seeing the need to be on time;
  Ensure students hand in assignments on time;
- · Finish class on time.

## Trustworthiness:

· Value of teacher expectations being met, eg students work well when left alone; students mark their own work; homework done for the benefits gained rather than fear of detention.

# **VALUES**

Some of the values which can be demonstrated and developed in mathematics are enumerated below. Suggestions about how the values can be highlighted are found in the following section of the framework which follows a sequence of mathematical topics alphabetically.

Acceptance of paradox

Accuracy

Aesthetic appeal

Appreciation of a Designer

Arranging priorities

Awareness of consequences

Awareness of potential

Awe

Balance

Care

Caution

Certainty

Choice

Courage

Dependability

Design economy

Development

Disciplined memory

Disposition to search

Economy of method

Economy of design

Economy of resources

Finding God

Following instructions

Informed decision-making

Inquiry

Investigation

Learning from mistakes

Logic

Mind expansion

Open mindedness

Order

Pattern

Perceptiveness

Personal responsibility

Positive peer selection

Positiveness

Postive use of difficulty and failure

**Practicality** 

Predictability

Progressiveness

Reasoning process

Reference to principles

Responsibility Self-worth

Sharing

Stewardship

Verification

Wise choice

Worth

# VALUES IN MATHEMATICS TOPICS

This section of the framework is designed to give teachers ideas about how mathematical values can be identified, highlighted and taught in classwork. The values are arranged alphabetically according to some of the major secondary school mathematical topics.

#### **ALGEBRA**

## Awareness of Consequences:

In equations the value that one substitutes for x results in certain consequences. This illustrates the influence of cause and effect in life.

#### Awareness of Potential:

As in asymptotes, we may become closer and closer to Christ in likeness but we never touch Him. Our relationship and potential to grow is continuous and infinite.

#### Balance:

Solving equations always requires them to be balanced. A satisfying Christian life always requires a balance. An unbalanced or messy sequence ends in the wrong answer.

#### Caution:

Test the solution of an equation to see if it works. Test things in life to see if they are worthwhile.

## Certainty:

Note the use of 'Xmas' instead of Christmas. Christ has been replaced by an unknown or variable quantity.

#### Choice:

Is an important part of mathematical reasoning. For example, we plot lines by choosing values for x and y. We choose between values such as speed and completing the task in detail or with accuracy. Many of our choices carry consequences, and we must learn what these are. The analogy holds for many life situations as well.

#### **Development:**

An example is that although the positive gradient of functions may vary, all are upwards.

#### Learning From Mistakes:

If you make a mistake, try to decide what went wrong and do not make the same mistake next time.

#### Order:

The transposition of equations is like the Christian life, for in life we cannot always see the immediate purpose in something. Sometimes things need to be re-ordered and sorted out to be useful. Formulae are based on order.

#### Positive Peer Selection:

Simplifying by collecting like terms has a parallel with collecting types of peer friends in life.

#### ALGEBRA (cont'd)

#### Positiveness:

When multiplying negative numbers, two negatives give a positive. God turns both positive and negative experiences into positive ones.

#### **ARITHMETIC**

#### Accuracy:

Implies economy because when we are accurate wastage is minimized. We should strive for care and neatness.

#### Awe:

Counting numbers as an infinite set, as God is an infinite being.

## **Economy:**

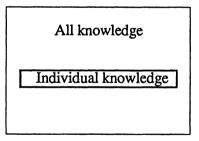
Mathematics should encourage efficient use of resources such as time, effort, space, materials. Economy includes efficiency in producing results. Clarity of expression is part of economy. Choosing the most effective alternative, involves values such as simplicity, conciseness, and clarity.

## Informed decision-making

When making choices, the greater the knowledge, the wiser the decision that can be made. Knowledge is founded on basic skills.

## Logic:

Note the venn diagram below on how knowledge relates to the existence of God:



The diagram shows that knowledge of the existence of God may exist outside individual knowledge.

#### Order of numbers:

This can be shown for example in: magic squares, number patterns, Pascal's triangle. Numbers and operations are ordered and wrong answers result if order is wrong.

## Place:

In mathematics, numbers have value according to their place (eg the 7 in 372 is worth 70). So in life, many things have value because of their place - place assigns value. We profit from knowing our rightful or appropriate place in many life situations.

#### Self-worth:

This can be shown by place value. For example the value of a digit is determined by its position in relation to the decimal point. Let God put the decimal point in your life.

We need arithmetic skills for many aspects of living. These influence:

- Personal independence
- Planned job opportunities
- Pleasure
- Progress
- Quality of life
- Self-esteem
- Self-preservation
- Service to society
- Stewardship of resources

Collectively the underlined values are summarized by <u>responsibility</u>, <u>practicality</u> and <u>wise</u> <u>choice</u>.

#### **CALCULUS**

## **Economy of Resources:**

This can be illustrated by using maximum and minimum values to calculate minimum material needed for maximum volume etc.

## Following instructions:

Is being willing to follow set guidelines and rules.

#### Inquiry:

Explores limits, and considers the infinite and the finite.

#### Logic in Reasoning:

Deductive and inductive reasoning are based on logic.

The results of logic are only as dependable as the truth of the original premise.

Two types of reasoning are deductive and inductive:

## Mind Expansion:

Calculus can be a tool available to attempt problems not solved by previous knowledge.

## Postive Use of Difficulty and Failure:

An example of this trait is as follows. The turning point in a graph occurs when  $f^{1}(x) = 0$ , so often the turning point in life occurs when we reach our lowest point.

### Reference To Principles:

Is our derivation of why we do what we do. Even though we do not always think every action through every time, we need to be aware of the basic original reason for doing what we do. In calculus we use first principles to explain why we follow a set method, then we just keep using that method knowing in the back of the mind why.

#### CONSUMER ARITHMETIC

## Arranging Priorities:

Money is <u>not</u> everything. We should be able to put money into its true perspective.

## Economy:

Refers to the ability to calculate values for wise spending and investing.

## Responsibility:

Refers to living within your means.

## Sharing:

We should develop the concept of planning to be able to help other people. We should not keep everything to ourselves.

## Stewardship:

Is budgeting, and responsible usage of funds (credit cards etc). The mathematics of consumerism often stresses the importance of stewardship when making decisions about purchasing goods. For example, students can be shown the importance of comparing prices and looking for the best buy.

#### Verification:

Is the ability to put something to the test, and to check its real value.

#### Wise Choice:

Is the ability to make informed intelligent decisions about spending.

#### Worth:

Demonstrates a practical application of mathematics, hence its value. Mathematics allows us to live as a useful citizen in our society.

#### **GEOMETRY**

#### Acceptance of Paradox:

A point is not really a point but a representation of a point. This concept is still very useful and important, and parallels our understanding of God.

## Appreciation of a Designer:

In engineering the strongest shape is a circle like the trunk of the tree. The tree, designed by God is everywhere in nature.

#### Design Economy:

The mathematics of the honeycomb shows the economy of design. Economy produces the greatest strength and volume from the smallest amount of material.

## GEOMETRY (cont'd)

## Logic:

By deductive reasoning and through observation of the world we can deduce a Creator's hand.

#### Reasoning Process:

Design in nature (logarithmic spiral of nautilus shell or honeycomb) can be used to support the argument for the existence of a Designer.

## Utility of Pattern:

The usefulness of pattern is illustrated as follows:

- Mathematics expresses the concept of pattern in nature.
- There is a high degree of dependability of pattern in nature.
- Pattern is a tool for observation, and a tool for analysis.
- Pattern is a means for prediction.
- Geometrical patterns are the building blocks for both technology and beauty.
- Number patterns are the basis of mathematical theories.
- Pattern in statistics enables prediction and forecast with a high level of dependability.
- Probability is sometimes observation of pattern.

#### **MEASUREMENT**

## Accuracy:

Accuracy is something we should always strive for. The limits to accuracy of results are dependent on the limits of the original measurements. We must recognize types of errors. Two types are: avoidable which include systematic errors such as parallel, transcription errors; and unavoidable which depend on factors such as the level of accuracy of measuring instruments. We may strive for accuracy, but we should recognize its limits.

#### Choice:

Is using correct and reasonable units of measurement, and a suitable level of accuracy for different situations.

#### Disciplined Memory:

Formulae must be learned. It takes effort to learn some things.

#### **Economy**

Is being able to calculate requirements that are needed, and to save wastage.

## Finding God

As with measurement where we do not need one hundred percent accuracy for it to work, we do not need to understand everything about God for Him to work for us.

## Open mindedness

We should not take the first or most obvious measurement for granted, but we should consider other possibilities before starting our problem solving.

## **Practicality:**

There is a practical use in measurement, yet there is not one hundred percent certainty or accuracy. Measurement is not absolute, but is only accurate to a certain point.

#### Verification

Is being able to measure for yourself, to double check and save being 'ripped off'.

#### Worth

Can demonstrate a practical application of maths, and shows that maths has value.

#### **PROBABILITY**

## Logic:

For example, there are too many factors and combinations of factors to make evolution probable. Also give gradations in examples of probability such as one in ten, one in fifty, one in a hundred, one in four million, Tatts Lotto.

## Perceptiveness:

Independent events do not necessarily affect each other. Superstitious people believe events are connected when they really are not. Psychological and supernatural factors can alter this relationship.

## Personal responsibility:

Probability is only a likelihood not a certainty. An example is death by cancer. Probability does not control or determine actual outcomes. It concerns groups not individuals. The larger the group, the more likelihood of the laws of probability operating. People get a false sense of security with probability. An example is when they consider car accidents or death by lung cancer.

## Personal responsibility:

Emphasize that life is not random.

#### Progressiveness:

It is possible to escape from probability. An example of an escape is Ron Halverson who made a socioeconomic escape.

#### Stewardship:

Explain why gambling is not productive — there is a gap between initiative and mathematical probability. The need for careful spending on insurance is based on probability. For example a non-drinker, non-smoker has cheaper insurance in companies such as ANSVAR. House insurance premiums are determined by location according to crime areas.

### Wise choice:

For example, with salvation there are two outcomes - saved or not saved. Emphasize that by choice you can increase the probability.

#### PROBLEM SOLVING ABILITY

The skill of problem solving is related to many mathematical values. Examples include the following:

## Acceptance of value tension:

When solving problems, we have to keep in mind competing values. Examples are speed of operation versus task completion.

## Anticipation:

To find minimum and maximum values when solving some problems, foresight is required. Anticipation pays dividends.

## Awareness of parameters:

Problem solutions often fall into certain parameters (for example time and cost). All the parameters have to be kept in mind constantly. When making decisions in life, it is also necessary to juggle parameters.

## Care in technique:

When solving problems, the technique is often as important as the answer. It can be important to carefully record the way we solve problems for future reference.

## Careful problem formulation:

Problems in mathematics and in life often occur simply because of the way they are formulated.

## Creativity:

We use different approaches to solve problems. We develop different "proofs" for mathematical theorems.

#### Order:

Algorithms have a basis in order.

## SEQUENCE AND SERIES

#### Acceptance of Paradox:

There are paradoxes in design. So in life apparent contradictions can be working together in a large scheme.

#### Aesthetic Appeal:

Golden ratios are pleasing to the eye.

## Dependability:

The value of dependability is shown in a sequence.

## Economy of Design:

Fibonacci Series occurs often in nature eg. 1, 1, 2, 3, 5, 8....suggesting an intelligent source for number patterns:

• Found in pine cones, sunflower, pineapples.

• It also indicates one source and mind rather than mere matter.

• Relates to Pascal's triangle, again reinforces the same thread of there being a pattern or design.

This indicates economy of design.

## Flexibility:

In sequences certain steps need not be reversed, while others must be able to be reversed. For example, the statement 'if and only if' is one that is true as it reads, and also in reverse. In life, some things are reversible, but some are not. We need to know the difference.

#### Order:

Sequences and series are orderly. Mathematics is a system that reflects order. Mathematical modelling works on the assumption that nature is orderly.

## **Predictability:**

Can be a support for design and a designer. It comes from working out formulae from patterns. This process relates to making intelligent choices.

#### TRIGONOMETRY

#### Awe:

Is a sense of wonder through a look at the infinite. We can use tan graphs.

#### Care:

Is diligence in assessing, interpreting, and analysing the information given to start with.

#### Choice:

Is selecting the correct ratio in the process of obtaining a correct result.

#### Courage:

Means willingness to experiment, to make a start even if you cannot see the end from the beginning.

#### Disposition to search:

There is no end to knowledge. For example, after angle sum and pythagoras, there is still more.

#### Investigation:

Is discovering real-life uses of trigonometry. Examples are navigation, astronomy, and surveying.

# TRIGONOMETRY (cont'd):

# Logic and Order:

Refer to making sure that statements follow the correct pattern, and are true.

## Pattern:

Shows us how the occurance of trigonometry graphs in life varies (light, sound, heartbeat, pulse rate etc).

# VALUE TEACHING STRATEGIES

There are a number of strategies that can be used to get across the values component of mathematics. Some of these are briefly explained below. It is our hope that as teachers think about the valuing process in mathematics, we will generate many more tactics of teaching values.

- 1. Choose content which lends itself to introducing values. An example is the topic of design in nature. See the appendix on design.
- 2. Use analogies, parallels, and object lessons. This technique is one of the most obvious ones, and one we have used heavily in our examples of valuing in the ten topic areas of this framework. The following example illustrates this tactic:
  - <u>Balance</u>: Solving equations always requires them to be balanced. So a satisfying Christian life always requires a balance. An unbalanced or messy sequence ends in the wrong answer, or in the wrong lifestyle.
- Use the history of mathematicss to illustrate biographical values in the lives of mathematicians.
- 4. Raise issues related to the nature of mathematics, or the usefulness of mathematics. Do this in discussion, as questions, or in challenges to students.
- 5. Identify values involved in problems and examples. An example is:
  - Stewardship is budgeting, and responsible usage of funds (credit cards etc).
- 6. Use comparison, particulary in topics such as statistics, to discover values. An example is:
  - <u>Predictability</u>: Comes from working our formulae from patterns. This process relates to making intelligent choice.
- 7. Use decision-making situations to make choices and solve problems. Refer to the consequences of decisions, and ways of verifying that these consequences will occur. See the section on problem solving in this framework.
- 8. Use evaluation to make value judgements about the worth of problem solutions or procedures we use.

## PROCESSES AND SKILLS

Set out below are a number of processes and skills that could be taught in secondary mathematics courses. The list is not exhaustive, and is meant to help teachers see at a glance a profile of skills that a student would try to develop over time.

#### **PROCESSES**

Approximation
Calculation
Communication
Data processing
Decision-making
Estimation
Exploration
Inquiry
Problem solving
Thinking

#### **SKILLS**

#### Calculation

Performing calculations Substituting Using a calculator Verifying results

#### Communication

Comparing
Comprehending
Describing
Explaining
Following instructions
Neatness
Representing
Setting out
Sketching
Terminology
Writing skills

## **Data Processing**

Classifying
Collating
Collecting data
Organizing information
Presenting
Recording
Summarizing

## General

Application
Constructing
Drawing
Graphing
Manipulating
Measuring
Mental arithmetic
Patterning
Plotting

Risk taking Using computers

Inquiry
Investigating
Listing
Multiple references
Seeking patterns

Problem Solving
Analyzing information
Looking ahead
Looking back
Problem discovery
Seeking information

#### Social

Synthesizing

Accepting responsibility Contributing Following directions Initiative Listening Persevering Tolerance

#### Thinking

Abstracting Analysizing Classifying Comparing Deducing Generalizing Inferring Synthesizing Validating

## ASSESSMENT

Assessment is the measurement of students' performance in relation to the aims and objectives of their courses.

Assessment can take many forms. Informal assessment is carried out through questioning and observing individual students as they work, while examinations and tests are examples of more formal means of assessment. The assessment should be on the full range of ability processes which include knowledge and skills and not just recall.

A range of assessment methods is needed as different methods are more appropriate for different students and certain methods favour particular students.

Assessment may be carried out for one or more of the following reasons:

- To find what existing knowledge or prior experience students bring to the learning task;
- To monitor the progress of students;
- To provide motivation;
- To provide feedback to students;
- To measure the extent to which students meet the course objectives;
- To establish a single global mark;
- To assess students' potential in the subject;
- To provide feedback to the teacher.

Assessment should take account of the teaching of values. Both the process and the content of valuing can be assessed. For example teachers can ask students to recall mathematicians' values or identify values in the examples set in tests and assignments. Students can also be asked to draw their own spiritual analogies, or they can show their ability to make decisions or make value judgments about procedures. When assessing, refer to the suggestions on teaching values, on solving problems, and to the examples of value statements.

The next assessment issue relates to the reporting of expectations and results. It is clear that students achieve better when learning expectations are spelled out clearly and regularly, when assignments are well structured, and when assessment results are provided promptly.

Evaluation extends beyond assessment of how well students are reaching objectives. It goes further in attempting to judge the merit of the course and its objectives, and it seeks ways to constantly improve instruction. Therefore some evaluation could be informal. Teachers may for example observe classroom signs of teaching success, interview students informally about the course, or ask them to evaluate the course in a written questionnaire. Good teachers enjoy their success, but keep a critical eye on their own performance.

Overall, evaluation requires teachers to critically think about how achievable their objectives are, how these objectives reflect school philosophy, how well students are mastering skills and concepts, and about the appropriateness of their assessment procedures.

# A TOPIC SUMMARY SHEET

Below is a summary of part of a mathematics topic. The summary is meant to illustrate how a teacher can use the framework document to bring together the key elements of the framework when thinking about how to be more systematic in dealing with values.

**Topic:** Statistics

### Content:

- Sampling
- Graphing
- Frequency Distribution
- · Mean, median, mode
- Standard deviation

## Practicality:

This topic aims to help students:

- Understand statistics to critically evaluate information presented to them. For example use advertising, surveys, political articles, church reports.
- To learn how to predict.
- To make intelligent decisions.

#### Values and Value Statements

- 1. <u>Honesty</u>: There is a danger of using quotes haphazardly 'out of context' or through distortion, as illustrated by the misleading use of a statement. The same holds for 'selective' listening.
- 2. <u>Wise choice</u>: Wise choices require sufficient and correct information.
- 3. <u>Truthful presentation</u> is illustrated by:
  - -not graphing in a misleading manner.
  - -placing figures in a correct context.
- 4. <u>Critical evaluation</u>: We should not take information at face value.
- 5. <u>Uniqueness</u>: We need to accept uniqueness, or sustain our unique self-image if we are different. Statistics tend to stereotype people when we are each unique 'average' 'normal'. Statistics compare individuals should we compare ourselves with the mean or with ourselves?
- 6. <u>Verification</u>: Statistics can clear up wrong perceptions.
- 7. <u>Positive Acceptance</u>. We cannot change some things so we have to accept them and make the best of them.
- 8. <u>Influence of Individuals</u>: If we take one value out of a set of scores, that affects the result. For example, to attain the mean, we sometimes remove the highest score. Each person has an effect on society, an effect that is not shown in means.
- 9. <u>Wise decision-making</u> requires an overall view of a situation. An example is gossip. We should not accept only what one person says plan to gather as much data as possible.
- 10. <u>Cautious Prediction</u>: When using previous information and experience to predict the future we must use caution. An example of the need for caution is Pascal's wager. Relate the size of the prize to the probability of winning a prize in a lottery etc.

## Strategies For Teaching Values:

- 1. Survey how choices are made.
- 2. Illustrate how statistics clear up misperceptions of products.
- 3. Collect examples of misleading statistics and examine the facts.

4. Show how percentages can be made to vary by using gross versus net pay.

5. Use statistics on a 'quit now' program and a tobacco advertising program and compare them.

6. Prepare a worksheet to show how taking mean, median, mode can lead to a different interpretation of statistics and incorrect decisions. We need to consider all.

7. Use a bell curve to illustrate the idea that on any issue we should expect extremes, a range of behaviours and beliefs.

8. Use a set of figures to show how selectivity and ignorance of contextual factors bias perception.

9. Relate uniqueness and extremes to bell curves and peer pressure.

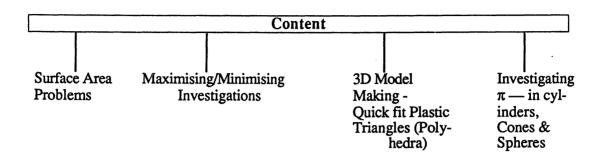
## **Processes**

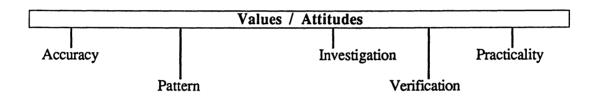
A wide range of processes are involved. Emphasize particularly:

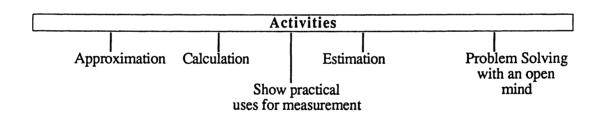
- Data processing
- Calculation
- Thinking
- Communication
- Inquiry

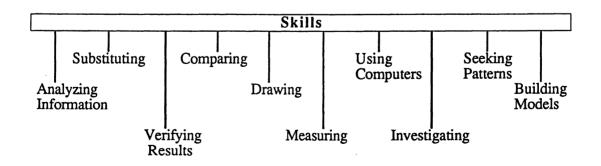
# A TOPIC OVERVIEW

## MEASUREMENT - SURFACE AREA









- Integration With Program -

# **CORRELATION CHARTS**

To make your overall planning more systematic, it could be helpful to use the correlation charts on the following pages. You can develop each chart to make an overview of the use of value-oriented concepts, ideas or skills in your course, unit or topic. These topics or units together comprise a course. We suggest that you could photocopy your own correlation charts from the ones here. When using the charts, write the title of your units vertically. These charts do not show all elements of the framework, yet they do show some key elements of the framework in condensed form. You could use them as an alternative to flowcharts. The main thing is to develop your own style of planning more systematically.

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## **APPENDICES**

# APPENDIX 1

# HOW MATHEMATICS SHOWS GOD'S WORK IN NATURE

The study of mathematics seems to point to the changlessness of God. It reveals the wisdom of God, and that He is a God of order and system.

The revelation of God in nature involves ideas of number, form, symmetry, system, and laws governing the existence and harmonious working of all things. Everywhere there are evidences of such mathematical relationships.

A proper understanding and appreciation of the works of God demands of man a knowledge of the underlying principles of mathematics. To this end God has led man to develop an efficient system of number, computation, and expression of natural laws.

Mathematics is a revelation of the thought life of God. It shows Him to be a God of system, order and accuracy. He can be depended upon, and His logic is certain. By thinking in mathematical terms, therefore, we are actually thinking God's thoughts after Him.

#### SPIRALS IN DESIGN

- 1. Spirals seem to be a favourite of God. He used them in designing many things in nature, such as:
  - The unfolding of a bud;
  - The chambered Nautilus shell;
  - Logarithmic spirals which are found in elephants' tusks, horns of wild sheep and even canaries' claws;
  - The spines of a pine cone going in opposite directions.

This phenomenon is more evidence of a Master Designer when it can be demonstrated that the spiral of a daisy has a ratio of 21:34 which just 'happens' to be the same ratio as two adjacent Fibonacci numbers which are generated by starting with 1 and adding the last 2 to arrive at the next, such as 1, 1, 2, 3, 5, 8, 13, 21, 34. The same ratio holds true in many other plants with a spiral leaf-grown pattern.

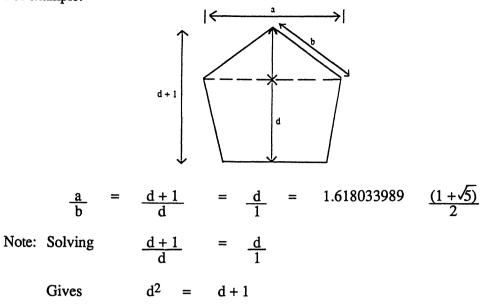
The chambered nautilus shell and the spirals of a pine cone both follow the Fibonacci sequence. The ratio between any Fibonacci numbers converges to the ratio 1:1.618 - called 'the golden ratio'.

If a rectangle is drawn around the human head and trunk the sides of the rectangle are in the Golden ratio. Some other parts of the body are also in this ratio.

2. Figures with rotational symmetry of order 5 are often the basis of nature's pattern.

There are many varieties of plants where flowers form regular pentagrams or five-pointed stars.

Some starfish are also formed in the shape of the pentagram. The pythagoreans studied the pentagram and noted that the ratio of several of its dimensions are the Golden Ratio. For example:



and hence, 1.618...x 1.618... = 2.618...

Dividing by 'd' gives d = 1 + 1/d so the reciprocal of 1.618... is 0.618...

This 'magic' irrational number, the Golden Ratio, is to be found in many apparently unconnected situations, including Classical architecture, the ratio of the numbers of spines in the spirals of pine cones going in opposite directions, and the distances between leaf attachment points on a branch.

- 3. Entire books have been written about the bee and how God has given this small insect the ability to build a honey comb using the smallest amount of material to enclose the greatest possible amount of honey. Mathematicians have used calculus to find these precise dimensions which the bee uses.
- 4. All are familiar with the endless variety and perfect symmetry that God has put into the tiny snow-flake. These endlessly embellished hexagons, never repeating themselves, show the infinite variety and creativity of our Master Designer.
- 5. Man-made objects placed under a microscope tend to show the flaws and imperfections that do not appear visible to the naked eye. The opposite is true of God's creations. The tiny diatom, a microscopic sea plant with the mosaic beauty of a stained glass window, is a classic example of what God's objects show when viewed under magnification. The higher the magnification, the greater the beauty, order, and symmetry.
- 6. The common geometric forms such as, squares, triangles, rectangles, and hexagons, appear to be the building blocks that God used in creating many things. An example of this is formed in viewing a cross section of a semiprecious gem called tourmaline which is mined in Madagascar. The stone's prismatic structure consists of a series of similar triangles neatly nested one inside the other.
- 7. God seems to delight in the creation of varied geometric shapes. Common mineral quartz for example, often occurs as a triagonal trapezohedron or more simply a crystal structure made up in a triangular arrangement with individual faces being four-sided.

Educational films are readily available that reflect the principles discussed above. These include films such as:

- · 'Donald in Mathmagic Land'
- 'The Mathematics of the Honeycomb'.

'<u>Donald in Mathmagic Land</u>' deals with mathematics as found in nature, music, games, and art. It shows very skilfully how mathematics is an integral part of the world in which we live.

'<u>The Mathematics of the Honeycomb</u>' graphically shows the seemingly unbelieveable use of mathematics that is made by the honey bee in both the construction of its honeycomb and in its navigational techniques.

In the words of Galileo: "Mathematics is the language that God used to create the universe."

# MAKING MATHEMATICS PRACTICAL AND RELEVANT

Conventional mathematics teaching has not only neglected and moved further away from the reality of mathematics in society, but it has substituted for the reality a lifeless 'skeleton' of abstract ideas and objectives which has become an end in themselves. The result of this for many students is that mathematics has become a difficult subject to teach, and one which holds little motivation for them.

If mathematics is going to be interesting and relevant for students, we need to concentrate on the ways students learn, and on the types of activities which are of interest to them. For learning to be relevant for junior-secondary pupils it should include the following characteristics:

- The course must have order and direction.
- Students' natural enthusiasm is to be utilized. Teachers should avoid curbing this
  enthusiasm.
- New ideas cannot be rushed. They need time to develop.
- Variety is the spice of life. A change in teaching content and method adds interest.
- Doing is more interesting than listening. Students need opportunity to manipulate materials.
- Most students like to do some drawing and colouring.
- Students like puzzles, games, and simulations.
- Teachers realise that all students will be at different starting points when they start new topics.
- · Most students enjoy group activities.
- Students are interested in a range of topics which they see as relating directly to their daily life expereinces.

#### **IMPLICATIONS**

Teachers should feel free to vary methods to suit the children and the topic. They should also break out of the traditional topic boundaries if they feel that the situation requires it. They should ask if their course is appropriate for preparing students for life, rather than whether it is appropriate only for the syllabus.

To succeed, teachers will have to find time to effectively use the resources which are available to them, both inside and outside the classroom. Also, the development of 'mathematics in society' type units which are both motivating and realistic, needs to be an ongoing task. Development of such units may need a theme, a group of teachers and time to brainstorm and plan.

Topics which are relevant to the life, interests, needs and goals of secondary students are listed below. The list is suggestive of possibilities, and is not prescriptive.

#### The Body

Examples: heart beat, breathing, health statistics, reaction times, medicine dosages.

#### **Business and Industry**

Examples: making a profit, small businesses, market research, automatic tellers.

#### Careers

Examples: nursing, computer programer, physics, electrician, architecture.

**Commission Sales** 

Examples: land agent, car dealers, auctions.

Community

Examples: elections, water rates, pollution levels, traffic flow, schools.

Energy

Examples: electricity, gas, oil, uranium.

**Entertainment** 

Examples: real costs, transport, special clothing, admission, crowd size, premises rental.

**Families** 

Examples: genetics, hair and eye colour, family tree.

**Farming** 

Examples: land size, yields, volumes, capacity, transportation, costs of production.

Food and Drink

Examples: kilojoules, diets, protein, bottle sizes.

**Hobbies** 

Examples: model cars and planes, painting, sewing, aerobics.

Home

Examples: running a car, furnishing the home, building a shed, recipes, house plans

**Machines** 

Examples: motors, pulleys, rockets, cameras, computers, lawn mowers.

Managing money

Examples: wages, budgeting, hire purchase, credit cards, insurance, investment.

Media

Examples: opinion polls, viewing audience, advertisements

**Outdoors** 

Examples: heights, distances, areas, perimeters, navigation, orienteering, estimation.

Phone

Examples: charges, ISD, Toll calls, time, codes.

**Population** 

Examples: births, deaths, marriages, migrants, urbanization.

Postage

Examples: rates, local, overseas, services.

Recreation

Examples: number patterns, tessellations, codes, nature, space, design.

School

Examples: growth rates, boys, girls, home suburbs, teacher/pupil activities, building

plans.

Solar System

Examples: planets, satellites, rockets, comets, speed of light.

**Sports** 

Examples: skateboard ramp, skiing, motor racing, yatching, playing fields, records.

**Tourism** 

Examples: accommodation, sightseeing, availability of services.

Transport

Examples: loads, buses, ships, planes, speed, types of vehicles.

Travel.

Examples: planning, distances, direction, timetables, 24 hour clocks, budgeting.

Weather

Examples: temperature, wind velocity and direction, rainfall.

# MAKING MATHEMATICS FUN

This framework holds the view that at times mathematics needs to be made fun for it to be motivating for students. Suggestions for making the subject fun to learn are listed below.

#### 1. Presentation of the classroom:

- Colourful stimulating posters, students work, formulae, topic starters eg "where do we use this - 'e', 'π'?"
- Periodically change your poster layout
- · Be prepared to change your seating layout

## 2. Games/puzzle corner:

• Give your students access to stimulating games and puzzles to use when they have finished set work or during breaks. eg use equable, chess, tangram puzzles, teasers, soma cube, mastermind etc

#### 3. Classroom activities:

- Mathsmind, championship, (mental arithmetic knockout competition carried out over a period of time)
- Line design eg string art (see the book 'Line Design')
- Model building eg polyhedra or nets (a cheap way is to use straws and putty)
- Use of the RIME activities
- Computer corner eg for those finished early
- Warmup or alternative teaching activities eggive all students a maze to solve
- Drawing/painting dynamics eg how a picture is composed using the golden ratio and intervals of one third
- Calculator activities and competitions

#### 4. Outdoor activities:

- Measurement eg measure and map the school or a measurement scavenger hunt
- Statistical survey
- Shopping and comparing prices (a good general level activity)
- Rates and time eg measure walking speed versus running etc
- Exploring building design for geometrical shapes
- Excursions with mathematics activities
- Finding the golden ratio in nature eg see national parks and zoos in some states

#### 5. Use of technology:

- Relevant videos and films
- OHP'S
- Slide sets and film strips
- Computer demonstrations

## 6. Use of concrete materials:

• To illustrate mathematical concepts (particulary in algebra) eg centi cubes, blocks, mira mirrors, matchsticks and plasticine etc

## 7. Teacher variables:

- Vary your teaching approach
  Use as many stimulus materials as possible
  Show your enthusiasm about your subject
  Be well prepared for any activity
  Try to cater for all levels with your activities
  Make sure your students are always busy

# PROBLEM SOLVING AND INVESTIGATION

#### **Definition:**

A problem is a situation in which an individual or group accepts the challenge of performing a task for which there is no immediately obvious way to determine a solution.

#### Rationale:

Problem Solving is not a content-based strand. Teachers should use problem solving as a technique to:

- Involve a group of students working as a unit;
- Involve discussion amongst students within the group;
- Involve comparison of group strategies;
- Stimulate interest in mathematics;
- Relate mathematics to real situations;
- Extend students intellectually.

# Skills Required:

Skills are the building blocks used in solving a problem. A listing of these skills is given below.

#### Problem Discovery, Formulation:

- State the problem in your own words.
- Clarify the problem through careful reading and by asking questions.
- Visualize an object from its drawing or description.
- · Follow written and/or oral directions.

#### Seeking Information:

- Collect data needed to solve the problem.
- Share data and results with other persons.
- Listen to persons who have relevant knowledge and experiences to share.
- Search printed matter for needed information.
- Make necessary measurements for obtaining a solution.
- · Record solution possibilities or attempts.
- Recall and list related information and knowledge.

## **Analyzing Information:**

- Eliminate extraneous information.
- Find likenesses and differences and make comparisons.
- · Classify objects or concepts.
- · Make and use a drawing or model.
- Make and/or use a systematic list or table.
- Make and/or use a graph.

- Look for patterns and/or properties.
- Use mathematical symbols to describe situations.
- Break a problem into manageable parts.

## Solve - Putting It Together - Synthesis:

- Make predictions, conjectures, and/or generalizations based upon data.
- · Make decisions based upon data.
- Make necessary computations needed for the solution.
- Determine limits and/or eliminate possibilities.
- · Make reasonable estimates.
- · Guess, check, and refine.
- Solve an easier but related problem. Study solution process for clues.
- Change a problem into one you can solve (Simplify the problem)
- Satisfy one condition at a time.
- Look at the problem situation from different points of view.
- Reason from what you already know (deduce).
- · Work backwards.
- Check calculated answers by making approximations.
- Detect and correct errors.
- Make necessary measurements for checking a solution.
- Identify problem situations in which a solution is not possible.
- Revise the conditions of a problem so a solution is possible.

## Looking Back - Consolidating Gains:

- Explain how you solved a problem.
- Make explanations based on data.
- Solve a problem using a different method.
- Find another answer when more than one is possible.
- Double check solutions by using some formal reasoning method (mathematical proof).
- Study the solution process.
- Find or invent other problems which can be solved by certain solution procedures.
- Generalize a problem solution so as to include other solutions.

# Looking Ahead - Formulating New Problems:

Create new problems by varying a given one.

#### Possible Instructional Approaches:

Different styles of teaching such as direct instruction, guided discovery, laboratory work, small group discussions, non-directive instruction and individual work all have a role to play in problem solving.

#### Some specific suggestions are:

- Set an example by solving problems and by sharing these experiences with the pupils.
- Reduce student anxiety by encouraging communication and cooperation. On frequent occasions problems might be investigated using a cooperative mode of instruction along with brain-storming sessions.
- Encourage pupils in their efforts to solve a problem by indicating that their strategies are worth trying and by providing them with sufficient time to investigate the problem; stress the value of the procedures pupils use.
- Use the pupils' ideas including their mistakes in solving problems and developing lessons.

- · Ask probing questions which make use of words and phrases such as:
  - I wonder if ...?
  - Do you suppose...?
  - What happens if...?
  - How could we find out ...?
  - Is it possible that...?
- Reinforce the asking of probing questions by pupils as they search for increased understanding. Pupils seldom are skilled at seeking probing questions but they can be taught to do so. If instruction is successful, questions of the type, 'What should I do now?,' will be addressed to themselves rather than to the teacher.

Generally, instruction should be non-directive, but at times suggestions may need to be given. If possible, these suggestions should be made in the form of alternatives to be explored rather than hints to be followed.

## Some Examples of Problem-Solving Strategies:

#### **GUESS AND CHECK:**

The Happy Holiday Hotel is blessed with cheerful bedbugs. In each single bed you can find 7 bedbugs and in each double bed there are 13 bedbugs. If there are 106 bedbugs altogether, how many double beds are there?

#### LOOK FOR A PATTERN:

What is the sum of the first:

- 4 odd numbers?
- 5 odd numbers?
- 7 odd numbers?
- 10 odd numbers?
- 3467 odd numbers?

#### MAKE A SYSTEMATIC LIST:

There are six basketball teams in a tournament. The teams are lettered A - F. Each team plays each of the other teams twice. How many games are played altogether?

#### DRAW A PICTURE:

A 12 cm by 16 cm rectangular piece of paper has a 2 cm by 2 cm square cut out of each corner. Then the sides are folded up to make an open box. Find the volume of the box.

#### **ELIMINATE POSSIBILITIES:**

In a certain bank the jobs of cashier, manager and teller are held by Brown, Jones and Smith, but not necessarily in that order.

- a. The teller, who was an only child, earns the least.
- b. Smith, who married Brown's sister, earns more than the manager.

#### Who has which job?

An awareness of the strategies being used to solve a problem is probably the most important step in the development of a pupil's problem-solving abilities.

# REMEDIAL MATHEMATICS

#### Rationale:

Because such a wide range of abilities and attitudes exist within a relatively small number of children a process of help must be set up to cater for such variations. In larger schools streaming may partially assist in this process.

#### **Definitions:**

A remedial student is one who has difficulty understanding mathematical concepts and doing elementary computational skills.

An under-achiever includes those who perform significantly below expected achievement commensurate with their age.

A high achiever is a student who performs beyond the expectations of the course.

#### Identification:

To identify students who need extra assistance, the following steps are possible:

- Testing all entrants into high school with standardized tests.
- Monitoring progress against expected performance.
- Retesting at senior high school.
- Ensuring access to student chart records particularly primary reports.
- Pre-testing, post-testing and diagnostic testing where appropriate.

Step 1 identifies the level at which any student is expected to perform, indicating remedial students.

In conjunction with Step 1, Step 2 identifies students who are under achievers.

The higher achiever generally becomes evident during Steps 1 and 2.

#### How to Assist:

- 1. Support teachers:
  - FTE
- 2. Support materials:
  - · graded work;
  - · diagnostic materials;
  - audio visual;
  - · computer based.
- 3. Pre-service training.
- 4. In-service training.

- 5. Utilize government bodies:

  - testing;materials;personnel.

# 6. Parents:

• supervise student activities at school and/or home.

# 7. Financial:

- governmentschool.

# ESSENTIAL COMPETENCIES

# Australia, Year 7 and New Zealand Form 2

#### Whole numbers:

- Read, write, name, and order whole numbers in general usage and relate to number line.
- Maintain recall of basic facts to 10 + 10 and multiplication to 10 x 10.
- Maintain addition and subtraction skills for four-digit numbers.

• Estimate skills at all stages in the basic operations.

• Solve word problems at all stages in the basic operations.

Multiply and divide by powers of 10.

• Identify the set of factors and multiples of a number.

• Graph given number pairs on coordinate plane (whole numbers only).

#### **Decimals**

• Name and order decimals to two decimal places and place on number line.

• Add and subtract three place decimals.

• Apply addition and subtraction to practical measuring problems, including money, shopping, budgeting, lengths and perimeter.

• Multiply and divide two place decimals by 10, 100, 1000.

• Multiply with two-digit multiplier and estimate product.

- Apply to money, length, in real life situations, eg. school trips. Apply to areas.
- Apply to metric measure conversions, eg. km-m, m-mm, kg-g, tonne-kg, L-mL.

#### **Fractions:**

- Understand fractions with denominators 2, 3, 4, 5, 10, 100.
- Use fractions to express:
  - part of a whole;
  - one whole number divided by another.
- Place fractions with common denominators on number line.
- State equivalent names for both whole numbers and common fractions, and express fractional numbers greater than 1 as mixed numerals.
- Write simple addition or subtraction sentences using fractions with same denominators of 2, 3, 4, 5, 10, 100
- Find by informal means a fraction of a whole number, denominators 2, 3, 4, 5, 10 (eg 2/3 of 12).

#### Percentage:

- State the meaning of per cent and convert from hundredths to percentage (percentages multiples of 10, 25).
- Find percentage of whole number (percentage a multiple of 10, 25).

#### **Integers:**

- Use integers to describe everyday situations such as temperature below zero, bank balances, game scores.
- Represent integers on number line. Order integers.

### Other operations:

• Use the convention for order of operations, eg  $3 + 4 \times 2 = 11$ .

## Mathematical language:

- Use the following terms appropriately: subset, (finite, infinite) set, empty set, union, intersection.
- Use set notation and symbols where appropriate.
- Use venn diagrams to show union, intersection of sets.
- Follow and interpret simple flow charts.

#### Measurement:

- Measure, using appropriate equipment:
  - · length to nearest km, m, cm, mm;
  - time, to nearest second;
  - mass (weight), to nearest kg, g;
  - capacity, to nearest l, ml;
  - · temperature.
- Construct and interpret temperature graphs.
- Read digital and analogue clock to nearest minute.
- Measure time interval.
- Measure in practical situations involving choice and use of appropriate units.
- Apply estimation skills to time, length, mass (weight) including the choice of appropriate units.
- Explore area of square, rectangle by discovery leading to formulae.
- Measure larger areas including land, using m squared, ha.
- Estimate and measure volume through practical work using cm cubed and m cubed.

#### Data and statistics:

- Read and interpret information presented in tabular form transport timetables.
- Collect and organise data and construct table.
- Collect data and construct bar graph and pictograph.
- Read above types of graph and form common sense conclusion from them.

#### Geometry

- Express compass directions as NSW etc, and use with maps and plans.
- Estimate angle size in degrees.
- Use a protractor to draw and measure angles to 360°.
- Identify in the environment:
  - parallel and perpendicular lines and planes.
  - vertical and horizontal lines and planes eg boxes, photographs, diagrams, quadrilaterals.

#### Shape:

- Identify triangles, quadrilaterals and polygons in the environment eg trapezium, equilateral triangle, hexagon, and regular polygons.
- Make these shapes; explore tessellations using regular polygons.
- Explore ways of covering surfaces, including tessellations. Investigate the regions of plane figures.
- Construct models of cubes, rectangular boxes, prisms, cylinders, pyramids, cones and regular polyhedra.
- · Recognise spheres.
- Name parts of circle, including circumferences, radius, diameter, and centre.

• Name parts of circle, including circumferences, radius, diameter, and centre.

# Transformations and symmetry:

- Recognise similar shapes and identify corresponding lengths and angles.
- Explore scale models, maps.
- Collect and draw examples of objects giving line symmetry.

The following competencies for school certificate level are in addition to the previous list which should be extended in the junior secondary school.

# **ESSENTIAL COMPETENCIES**

#### YEAR 10

(Without calculator use unless specifically stated)

#### Whole Numbers:

• Division up to 12.

#### Decimals:

- Addition, subtraction, multiplication (with a whole number), division (with whole number division).
- · Round off to at least two decimal places.
- Convert simple decimals to/from simple fractions, ie denominators of 2, 3, 4, 5, 10, 100.

#### Fractions:

- Addition and subtraction, with like denominators and simple denominators if not like.
- Multiplication and division with simple numbers.

#### **Percentages:**

Convert percentages to fractions and decimals and vice versa.

#### Measurement:

- Measure, draw and name angles.
- Measure and calculate from formula the perimeter and area of rectangles, triangles and circles.
- Use and apply the pythagoras theorem.
- Read and draw simple maps and scale drawings.
- Classify triangles.
- Know the basic metric units of length, area, capacity, time, mass and simple conversions.

#### Calculator:

• Use of the following keys: +, -, x, division, square root, x squared, MR, M+, Min, C, CA.

# **Estimation:**

- Use to check accuracy of answers.
- Best value for money in the supermarket.

## Basic formulae:

• Ability to use, ie  $A = 1 \times b$ , S = D/T.

## Ordered pairs:

• eg map reading.

# Line graph:

• Draw and interpret.

# Probability:

• Investigate in simple situations (eg tossing dice) including probabilities of 0 and 1.

# Problem solving:

- Basic strategies, ie guess and check, look for a pattern, make a systematic list, make and use a drawing or model, eliminate possibilities.
- Apply negative numbers to real life situations.

# MATHEMATICS DEPARTMENTAL POLICY

#### Preamble:

A Mathematics Department with a formal policy should be established in all Adventist High Schools for the following reasons:

- To develop and accumulate resource materials in a constructive manner;
- To institute consistent procedure for administration and evaluation;
- To approve useful introductory documents for new teachers;
- To improve communication among teachers, parents and students;
- To clearly state the philosophy of the department and relate it to the general Adventist Education philosophy.

For an example of some of the areas that should be developed in a Mathematics Department Policy see the Sydney Adventist High School Department Handbook index on the following pages.

## SYDNEY ADVENTIST HIGH SCHOOL

#### DEPARTMENT OF MATHEMATICS

#### **TEACHING HANDBOOK**

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